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The method of asymptotic analogies in the mass and heat transfer theory and chemical engineering science

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Abstract—A simple technique is suggested for constructing approximate relations with wide ranges of applicability (the same formula can be employed to describe a variety of qualitatively identical problems which differ geometrically, i.e. in the surface shape, flow pattern, etc.). The technique is based on the transition from ordinary dimensionless variables to special asymptotic coordinates. The illustration is made by reference to numerous specific problems in the theories of mass and heat transfer and physicochemical hydrodynamics. Comparison of the relations obtained with a number of typical cases, for which exact, numerical, approximate and asymptotic results are already available, shows a good accuracy and great capabilities of the method. The method can also be used with success in other fields of chemical engineering science, mechanics and physics.

1. DESCRIPTION OF THE METHOD OF ASYMPTOTIC ANALOGIES

SUPPOSE there is a certain class of problems which differ only in their geometric characteristics and which depend on the dimensionless parameter τ ($0 \leqslant \tau \leqslant \infty$). It is also assumed that for some specific (simplest) geometry the dependence of the basic quantity sought, x, on the parameter τ is known

$$x = F(\tau) \tag{1}$$

where F is a monotonous function.

Expression (1) will be employed as the basis for estimating the values of x which will correspond to the solution of other problems of the class considered. For this, equation (1) will be transformed in the following fashion.

Let the leading terms in the asymptotic expansions of the quantity x at small and large values of the parameter τ have the form

$$x \to x_0 \quad \text{for } \tau \to 0 \tag{2}$$

$$x \to x_\infty \quad \text{for } \tau \to \infty \tag{3}$$

where x_0 and x_∞ depend on τ in some known manner:

$$x_0 = \varphi(\tau), \quad x_\infty = \psi(\tau). \tag{4}$$

They can be found from an analysis of equation (1).

It will be assumed further that the requirement $\varphi/\psi \neq$ const. is met.

Using equations (1) and (4), two relations can be written

$$\frac{x}{x_0} = \frac{F(\tau)}{\varphi(\tau)}, \quad \frac{x_\infty}{x_0} = \frac{\psi(\tau)}{\varphi(\tau)}. \tag{5}$$

By expressing the parameter τ from the second equation and substituting it into the first equality (5), it is possible to find the explicit form of the function x in terms of the asymptotics x_0 and x_∞. As follows from equations (5), for the general case the structure of this relation is

$$\frac{x}{x_0} = f\left(\frac{x_\infty}{x_0}\right). \tag{6}$$

It is clear that in contrast to the original expression (1), equation (6) is unaffected by the technique of determining the dimensionless quantity x. Further it will be assumed that the variation region of each of the ratios x/x_0 and x_∞/x_0 is identical for all the problems of the class in question. The quantities x/x_0 and x_∞/x_0 will be referred to as asymptotic coordinates.

The asymptotic analogy method is that equation (6) is used to approximately calculate analogous characteristics for already rather a wide range of problems that describe qualitatively identical phenomena or processes differing only geometrically. For this purpose, having constructed relation (6), with the aid of equation (1), for some specific (for instance, the simplest) case, the procedure of calculating the magnitude of x for another problem of the same class is reduced to the determination of its asymptotics (in the same limiting cases as in equations (2) and (3)) with their subsequent substitution into equation (6).

It is important to note that the approximate relations derived by the above method yield the exact asymptotic result in both the limiting cases for $\tau \to 0$ and $\tau \to \infty$.

It should also be noted that the same ideas based on the use of asymptotic coordinates were employed

NOMENCLATURE

a	characteristic dimension of particle and droplet (radius of spherical particle)
C	concentration in flow
C_s	concentration on phase interface
C_∞	unperturbed concentration in incident flow
C_0	concentration at initial time instant
c	dimensionless concentration, $(C_\infty - C)/C_\infty$
$\bar{c}$	dimensionless concentration, C/C_s
$\tilde{c}$	dimensionless concentration, $(C_0 - C)/(C_0 - C_s)$
D	diffusion coefficient
K_s	surface chemical reaction rate constant ($W_s = K_s F_s$ is the surface reaction rate)
K_v	volumetric chemical reaction rate constant
k_s	dimensional constant of κ-order surface chemical reaction rate, $aK_s C_\infty^{\kappa-1}/D$
k_v	dimensionless constant of first-order volumetric chemical reaction rate, $a^2 K_v/D$
$2l$	cylinder length
Pe	Peclet number, aU/D
R	cylinder radius
$2R_1, 2R_2, 2R_3$	lengths of parallelepiped sides
Re	Reynolds number, aU/ν
r, θ, φ	spherical coordinate system fixed with respect to particle
S	dimensionless body surface area
S_*	body surface area
T	dimensionless temperature, $(T_0 - T_*)/(T_0 - T_s)$
T_*	temperature
T_0	temperature at initial time instant
T_s	body surface temperature
$\langle T \rangle$	dimensionless mean bulk temperature, $(1/V)\int_v T\,dv$
t	time
U	characteristic flow velocity
U_∞	unperturbed velocity of translational flow far from particle
V	dimensionless body volume
V_*	body volume
$\mathbf{v}$	dimensionless fluid velocity vector
y	dimensionless radial coordinate, r/a.

Greek symbols

β	ratio of dynamic viscosities of droplet and surrounding fluid ($\beta = 0$ corresponds to gas bubble)
Γ	body surface
Δ	Laplace operator
κ	order of surface chemical reaction
ν	kinematic viscosity of fluid
ξ	dimensionless coordinate normal to body surface
τ	dimensionless time ($t\chi/a^2$ in heat transfer problems and tD/a^2 in mass transfer problems)
χ	thermal diffusivity.

in refs. [1, 2] to improve approximate two-parametric dependent formulae.

A further comparison of the equations obtained by the asymptotic analogy method with a variety of specific cases, for which exact, numerical and approximate results are already available, shows good accuracy and great capabilities of the method. This is due to the fact that the final functional connection (equation (6)) of the quantity x, which is of interest to us, with its asymptotic remains the same (more precisely, varies little) for rather a wide range of identical problems, and the specific modifications and geometric differences (the shape and type of the interface and also the flow pattern at a distance from it) of these problems are rather completely taken into account by the corresponding asymptotic parameters such as x_0 and x_∞. In other words, the range of validity of the final expression (6) appears to be appreciably wider than that of the original relation (1). In this sense it can be said that the formulae of type (6) (as opposed to the original expression (1)) are more informative.

It should be noted that the values of x_0 and x_∞ can be obtained both theoretically and experimentally.

In the cases with $\psi/\varphi = \text{const.}$, the corresponding two-term asymptotic expansions of x for $\tau \to 0$ or $\tau \to \infty$ can be taken as the variables x_0 and x_∞ in equations (4).

Attention is now turned to the power-law dependence of asymptotics (4) which are most frequently encountered in the mass and heat transfer theory and in physicochemical hydrodynamics [2–6]

$$x_0 = A\tau^n (\tau \to 0), \quad x_\infty = B\tau^m (\tau \to \infty) \qquad (7)$$

where A, B and n, m are certain constants; $n \neq m$.

In this case the functions entering into equations (4) have the form $\varphi = A\tau^n$, $\psi = B\tau^m$. Placing these formulae into equations (5) yields

$$\frac{x}{x_0} = \frac{F(\tau)}{A\tau^n}, \quad \frac{x_\infty}{x_0} = \frac{B}{A}\tau^{m-n}.$$

Elimination of the parameter τ will give the sought relation

$$\frac{x}{x_0} = \frac{1}{A}\left(\frac{A}{B}\frac{x_\infty}{x_0}\right)^{n/(n-m)} F\left(\left(\frac{A}{B}\frac{x_\infty}{x_0}\right)^{n/(m-n)}\right). \qquad (8)$$

Equation (6) can be presented in the equivalent form

$$x/x_\infty = h(x_\infty/x_0)$$

where $h(z) = f(z)/z$, whilst equation (8) can be written as

$$\frac{x}{x_\infty} = \frac{1}{B}\left(\frac{A}{B}\frac{x_\infty}{x_0}\right)^{m/(n-m)} F\left(\left(\frac{A}{B}\frac{x_\infty}{x_0}\right)^{1/(m-n)}\right). \quad (9)$$

It should be noted that, for the power-law form of the asymptotics x_0 and x_∞, the four-parametric set of the quantities $x = a\tau^\alpha F(b\tau^\beta)$ in the τ–x plane (where $a, b, \beta > 0$; $\alpha \geqslant 0$; F is the prescribed function) passes over into the only curve in the x_∞/x_0–x/x_0 plane.

The case of the exponential dependence of the asymptotics x_∞ on the parameter τ will be considered later in Section 10.

Remark. In some cases the original relationship between x and τ can be specified in the implicit form

$$G(x,\tau) = 0. \quad (10)$$

Under the assumption that the leading terms of the expansions of the quantity x for $\tau \to 0$ and $\tau \to \infty$ are given by equations (4), equation (10) can be rewritten as follows:

$$G\left(\frac{x}{x_0}\varphi(\tau), \tau\right) = 0, \quad \frac{x_\infty}{x_0} = \frac{\psi(\tau)}{\varphi(\tau)}. \quad (11)$$

Eliminating the parameter τ from the second equation (11) and substituting it into the first equation will yield the relation between the complexes x/x_0 and x_∞/x_0.

For the power-law types of asymptotics (7), the sought relation has the form

$$G\left(A\frac{x}{x_0}\left(\frac{A}{B}\frac{x_\infty}{x_0}\right)^{n/(m-n)}, \left(\frac{A}{B}\frac{x_\infty}{x_0}\right)^{1/(m-n)}\right) = 0. \quad (12)$$

This equation will be needed later in Section 8.

Now the great capabilities of the asymptotic analogy method will be demonstrated on the examples of the problems of mass and heat transfer and physicochemical hydrodynamics.

2. THERMAL CONDUCTIVITY OF SOLID BODIES OF COMPLICATED SHAPE

Consideration will be given to the internal problems of unsteady heat exchange of differently shaped convex bodies with the surrounding medium. It will be assumed that at the initial time instant $t = 0$ the body temperature is uniform and equal to T_0, whilst for $t > 0$ the temperature on the body surface Γ is maintained constant and equal to T_s. In terms of dimensionless variables, the temperature distribution within the body is described by the following equation and initial and boundary conditions:

$$\frac{\partial T}{\partial \tau} = \Delta T \quad (13)$$

$$\tau = 0, \quad T = 0; \quad \mathbf{y} \in \Gamma, \quad T = 1$$

$$T = \frac{T_0 - T_*}{T_0 - T_s}, \quad \tau = \frac{\chi t}{a^2}, \quad \mathbf{y} = \frac{\mathbf{r}}{a} \quad (14)$$

where T_* is the temperature, χ the thermal diffusivity, a the characteristic body dimension, and $\mathbf{r}$ the radius vector of the corresponding coordinate system.

Here, attention will be mostly paid to the investigation of the mean temperature of the body $\langle T \rangle$ determined as

$$\langle T \rangle = \frac{1}{V}\int_v T\,\mathrm{d}v \quad (15)$$

where

$$V = \int_v \mathrm{d}v$$

is the dimensionless volume of the body.

To construct the approximate time dependence of the mean body temperature, the asymptotic analogy method will be employed. It is convenient that as the original simplest use will be made of the one-dimensional (with respect to space coordinates) problem concerning the heat transfer of a sphere of radius a. The solution of this problem is well known [7] and leads to the following expression for the mean temperature:

$$\langle T \rangle = 1 - \frac{6}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp(-\pi^2 n^2 \tau). \quad (16)$$

The asymptotics of equation (16) for short and long times have the form

$$\langle T \rangle_0 = 6\pi^{-1/2}\sqrt{\tau}(\tau \to 0), \quad \langle T \rangle_\infty = 1(\tau \to \infty) \quad (17)$$

and represent a specific case of equation (7) at $x_0 \equiv \langle T \rangle_0$ and $x_\infty \equiv \langle T \rangle_\infty$ where $A = 6\pi^{-1/2}$, $B = 1$, $n = 1/2$, $m = 0$. Substituting these values into equation (9) where $F = \langle T \rangle$, relation (16) can be rewritten as

$$\frac{\langle T \rangle}{\langle T \rangle_\infty} = 1 - \frac{6}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left[-\frac{\pi^3}{36}n^2\left(\frac{\langle T \rangle_0}{\langle T \rangle_\infty}\right)^2\right]. \quad (18)$$

In accordance with the asymptotic analogy method, equation (18) will be used for predicting the mean temperature of non-spherical bodies. To this end, for a body of prescribed shape the mean temperature asymptotics should be first calculated at short and long time intervals and thereupon substituted into expression (18).

When $\tau \to \infty$, the solution to the problem, equations (13) and (14), for a finite arbitrarily shaped body tends to the limiting value (equal to unity) which is determined by the boundary condition on the body surface. Setting $T = 1$ in equation (15), the asymptotics for the mean temperature at large values of τ can be found

$$\langle T\rangle_\infty = 1. \tag{19}$$

Next, consider the initial stage of the process corresponding to low values of dimensionless time. Equation (13) will be integrated over the body volume V. Allowing for the identity $\Delta T = \mathrm{div}\,(\mathrm{grad}\, T)$, the change-over will be made on the right-hand side of the resulting equation obtained from the volumetric to surface integral via the Ostrogradskiy–Gauss formula. This gives

$$\frac{\partial}{\partial\tau}\int_v T\,\mathrm{d}v = -\int_\Gamma \frac{\partial T}{\partial\xi}\,\mathrm{d}\Gamma \tag{20}$$

where the coordinate ξ runs inwards normal to the body surface.

At small times the temperature varies mostly in a thin zone adjacent to the body surface. In this region the derivatives along the body surface can be neglected as compared with the derivatives along the normal. Therefore, the temperature distribution for $\tau \to 0$ is governed by the following equation with initial and boundary conditions:

$$\frac{\partial T}{\partial\tau} = \frac{\partial^2 T}{\partial\xi^2};\quad \tau = 0,\quad T = 0;\quad \xi = 0,\quad T = 1 \tag{21}$$

where $\xi = 0$ corresponds to the body surface.

The solution of problem (21) is expressed in terms of the complementary error function

$$T = \mathrm{Erfc}\left(\frac{\xi}{2\sqrt{\tau}}\right). \tag{22}$$

By differentiating this equation with respect to ξ and setting $\xi = 0$, the local heat flux on the body surface can be found for $\tau \to 0$

$$\left(\frac{\partial T}{\partial\xi}\right)_\Gamma = -\frac{1}{\sqrt{(\pi\tau)}}. \tag{23}$$

Substituting equation (23) into equality (20) and integrating gives

$$\frac{\partial}{\partial\tau}\int_v T\,\mathrm{d}v = \frac{1}{\sqrt{(\pi\tau)}}S \tag{24}$$

where S is the dimensionless body surface area.

Now, both sides of equation (24) will be integrated over τ from 0 to τ. In view of the initial condition in (14), the sought asymptotic expression for the mean temperature can be obtained with $\tau \to 0$

$$\langle T\rangle_0 = 2\frac{S}{V}\sqrt{\left(\frac{\tau}{\pi}\right)}. \tag{25}$$

Placing equations (19) and (25) into equation (18) will yield the approximate time dependence of the mean temperature of an arbitrarily shaped body

$$\langle T\rangle = -1 - \frac{6}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left(-\frac{\pi^2 n^2 S^2}{9V^2}\tau\right).$$

This expression can be rewritten as

$$\langle T\rangle = 1 - \frac{6}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left(-\frac{\pi^2}{9}n^2\frac{S_*^2\chi t}{V_*^2}\right) \tag{26}$$

where S_* and V_* are the dimensional surface area and volume of the body.

For practical computations, it is expedient to use, instead of an infinite series, the simpler equation

$$\langle T\rangle = (1-\mathrm{e}^{-1.27\omega})^{1/2} + 0.6(\mathrm{e}^{-1.5\omega} - \mathrm{e}^{-1.1\omega})$$

$$\omega = \frac{S_*\chi t}{V_*^2} \tag{27}$$

the maximum difference of which from equation (26) is less than 2%.

Next, approximate relation (26) will be contrasted with the known accurate results obtained for heat transfer of non-spherical bodies.

First, a parallelepiped with the sides $2R_1$, $2R_2$ and $2R_3$ will be considered. The solution for the corresponding three-dimensional problem, equations (13) and (14), is constructed by separating the variables. It leads to the following equation for the mean temperature [7]:

$$\langle T\rangle = 1 - \left(\frac{8}{\pi^2}\right)^3 \times \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{k=1}^{\infty}\frac{1}{(2n-1)^2(2m-1)^2(2k-1)^2} \times \exp\left\{-\frac{\pi^2}{4}\left[\frac{(2n-1)^2}{R_1^2} + \frac{(2m-1)^2}{R_2^2} + \frac{(2k-1)^2}{R_3^2}\right]\chi t\right\}. \tag{28}$$

Taking into account that the parallelepiped surface area and volume are determined, respectively, as $S_* = 8(R_1R_2 + R_1R_3 + R_2R_3)$ and $V_* = 8R_1R_2R_3$ expression (28) can be written in the form

$$\langle T\rangle = 1 - \left(\frac{8}{\pi^2}\right) \times \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{k=1}^{\infty}\frac{1}{(2n-1)^2(2m-1)^2(2k-1)^2} \times \exp\left\{-\frac{\pi^2}{4}\frac{\left(\frac{2n-1}{R_1}\right)^2 + \left(\frac{2m-1}{R_2}\right)^2 + \left(\frac{2k-1}{R_3}\right)^2}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^2} \times \frac{S_*\chi t}{V_*^2}\right\}. \tag{29}$$

Table 1 (calculations were performed by L. Yu. Yerokhin) presents the results of comparison between the approximate (26) and exact (29) mean temperatures of the parallelepiped at six different values of R_1, R_2, R_3. It is seen that the maximum error

Table 1. Comparison of the exact and appropriate values of the mean temperature $\langle T\rangle$ of differently shaped bodies

Dimensionless time, $S_*^2\chi t/V_*^2$	0.05	0.1	0.2	0.3	0.5	1.0	1.5	2.0
Sphere, formula (26)	0.236	0.323	0.438	0.518	0.631	0.795	0.882	0.932
Approximate formula (27)	0.237	0.324	0.437	0.514	0.623	0.782	0.870	0.923
Parallelepiped, formula (29); $E_i = R_i/R_1$								
$E_2 = 1,\ E_3 = 0.25$	0.237	0.326	0.443	0.527	0.647	0.821	0.907	0.951
$E_2 = 1,\ E_3 = 0.5$	0.233	0.318	0.429	0.506	0.615	0.774	0.862	0.915
$E_2 = 1,\ E_3 = 1$	0.232	0.316	0.425	0.499	0.604	0.757	0.843	0.897
$E_2 = 1,\ E_3 = 2$	0.232	0.318	0.427	0.503	0.610	0.767	0.854	0.920
$E_2 = 1,\ E_3 = 4$	0.234	0.320	0.432	0.510	0.620	0.782	0.871	0.952
$E_2 = 2,\ E_3 = 4$	0.234	0.321	0.435	0.514	0.628	0.794	0.882	0.932
Cylinder, formula (31); $E = R/l$								
$E = 0.25$	0.236	0.325	0.440	0.522	0.638	0.807	0.894	0.942
$E = 0.5$	0.234	0.321	0.434	0.513	0.624	0.787	0.875	0.926
$E = 1$	0.233	0.319	0.429	0.506	0.613	0.770	0.857	0.910
$E = 2$	0.234	0.320	0.431	0.509	0.619	0.780	0.868	0.920
$E = 4$	0.237	0.326	0.444	0.528	0.649	0.823	0.909	0.952

of equations (26) and (27) for $0.25 \leqslant R_3/R_1 \leqslant 4.0$, $R_2/R_1 = 1$ is as much as about 5%.

Now consider the heat transfer of a finite cylinder. Let the cylinder radius be R and the length $2l$. The solution of the problem, equations (13) and (14), leads in this case to the following expression for the mean temperature [7]:

$$\langle T\rangle = 1-\frac{32}{\pi^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{\mu_n^2(2m-1)^2}\times\exp\left\{-\left[\frac{\mu_n^2}{R^2}+\frac{\pi^2(2m-1)^2}{4l^2}\right]\chi t\right\} \quad (30)$$

where μ_n are the roots of the zero kind Bessel function: $J_0(\mu_n) = 0$ (the values of the first 60 roots of μ_n can be found in ref. [8]).

Equation (30) can be rewritten as

$$\langle T\rangle = 1-\frac{32}{\pi^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{\mu_n^2(2m-1)^2}\times\exp\left\{-\frac{4l^2\mu_n^2+\pi^2R^2(2m-1)^2}{4(R+2l)^2}\frac{S_*^2\chi t}{V_*^2}\right\} \quad (31)$$

where $S_* = 2\pi R(R+2l)$ is the cylinder surface area and $V_* = 2\pi R^2 l$ the cylinder volume.

The computational results obtained for different values of the cylinder characteristic dimensions using exact (31) and approximate (26) relations are listed in Table 1. It is seen that the maximum error of equation (26) for $0.25 \leqslant R/l \leqslant 4.0$ amounts to about 3.5%.

3. MASS AND HEAT EXCHANGE OF AN ARBITRARILY SHAPED PARTICLE WITH A TRANSLATIONAL STOKES FLOW

Interest will be centred on a solid particle mass and heat transfer with a laminar translational viscous incompressible liquid flow. It is assumed that concentration far from the particle is constant and equal to C_∞, and that complete absorption of the substance dissolved in the liquid occurs on the interface. Let the particle surface Γ in the spherical coordinate system r, θ, φ be prescribed by the relation $r = r_s(\theta, \varphi)$ where r_s is the known function of θ and φ.

The concentration distribution C in the flow is represented by the convective mass transfer equation and the boundary conditions

$$Pe\,(\mathbf{v}\cdot\nabla)c = \Delta c \quad (32)$$

$$y = y_s(\theta,\varphi),\quad c = 1;\quad y\to\infty, c\to 0 \quad (33)$$

where $c = (C_\infty - C)/C_\infty$, $Pe = aU_\infty/D$, $y = r/a$, $y_s = r_s/a$, U_∞ is the liquid velocity at a distance from the particle, D the diffusion coefficient, a the characteristic particle size (which is usually represented by the radius of the equivalent volume sphere) and Pe the Peclet number. The distribution of the liquid velocity $\mathbf{v}$ is assumed to be known from the solution of the corresponding hydrodynamic problem concerning the flow around a particle.

The basic quantity of practical interest is the mean Sherwood number which is calculated from

$$Sh = \frac{I}{S};\quad I = -\iint_\Gamma \frac{\partial c}{\partial \xi}\,\mathrm{d}\Gamma,\quad S = \iint_\Gamma \mathrm{d}\Gamma. \quad (34)$$

For spherical particles it is possible, in lieu of equations (34), to write

$$Sh = -\frac{1}{4\pi}\int_0^\pi\int_0^{2\pi}\sin\theta\left(\frac{\partial c}{\partial y}\right)_{y=1}\mathrm{d}\varphi\,\mathrm{d}\theta. \quad (35)$$

Now, with the use of the method suggested, the approximate equation for calculating the Sherwood number per arbitrarily shaped solid particle in the translational Stokes flow is derived. This kind of flow complies with the limiting case $Re\to 0$; $Re = aU_\infty/\nu$ is the Reynolds number and ν the kinematic viscosity coefficient.

First the simplest problem of the mass transfer of a spherical particle will be considered which corresponds to the constant value $y_s = 1$ in the first boundary condition (33). (The liquid velocity field is well known in this case [2–6].) For predicting the mean Sherwood number per spherical particle, it is convenient to employ the approximate relation [5]

$$Sh = 0.5+(0.125+0.243Pe)^{1/3}. \quad (36)$$

This equation differs from the available data [5, 6, 9, 10] by about 2% over the entire range of Peclet numbers.

Now equation (36) will be transformed following the technique given in Section 1. For this, account will be taken of the fact that the asymptotics of Sh at small and large Pe have the form

$$Sh_0 = 1\ (Pe \to 0); \quad Sh_\infty = 0.624Pe^{1/3}\ (Pe \to \infty). \quad (37)$$

Equations (37) coincide with equations (7) accurate to the evident translation into the new notations $x \to Sh$, $\tau \to Pe$ at $A = 1$; $B = 0.624$; $n = 0$; $m = 1/3$. By substituting these values into equation (8), the sought relation is obtained

$$Sh = 0.5Sh_0+(0.125Sh_0^3+Sh_\infty^3)^{1/3}. \quad (38)$$

This equation can be used to predict the mean Sherwood number per non-spherical particle in the translational Stokes flow. As the auxiliary quantities Sh_0 and Sh_∞, the leading terms of the mean Sherwood number asymptotic expansions should be selected at small and large Peclet numbers, respectively. Some specific expressions for Sh_0 and Sh_∞ obtained theoretically and experimentally for particles of various shapes are given in refs. [4, 5]. In the particular case of an ellipsoid of revolution with the axis running along the flow the substitution of the corresponding values of Sh_0 and Sh_∞ into equation (38) yields the empirical equation suggested in ref. [5].

In ref. [10], the approximate problem of mass exchange between an ellipsoidal particle and the translational Stokes flow was investigated by the finite-difference numerical method. Two cases were analysed—when the length of the particle semiaxis oriented along the flow was five times larger and five times smaller than that oriented transverse to the flow. It follows from the results of numerical solution [10] and of comparative analysis [5] that the maximum error of equation (38) for an ellipsoidal particle in the above cases does not exceed 10%.

4. MASS AND HEAT EXCHANGE OF A SPHERICAL PARTICLE WITH A TRANSLATIONAL FLOW AT INTERMEDIATE REYNOLDS NUMBERS

It will be shown that equation (38) can also be used with success for predicting the mean Sherwood number per particle at intermediate Reynolds numbers. (Recall that the original relation (36) was obtained in the Stokes approximation, i.e. for $Re \to 0$.) Here the asymptotic Sh_∞ entering into equation (38) is proportional to $Pe^{1/3}$ [4] and depends on the Reynolds number

$$Sh_\infty = \sigma(Re)\,Pe^{1/3}. \quad (39)$$

The value of the parameter σ at different Re can be found by making use of the formula

$$Sh = 0.5+0.527Re^{0.077}(1+2Pe)^{1/3} \quad (40)$$

which was suggested in ref. [5] on the basis of processing the results of numerical solution for the problem on convective mass transfer of a spherical particle in the translational flow with $1 \leqslant Re \leqslant 200$ and $1 \leqslant Pe \leqslant 2000$. In the above range of Reynolds and Peclet numbers the maximum error of equation (40) is about 3%.

Equating the right-hand sides of equations (39) and (40) at $Pe = 2000$ yields the relation $\sigma = \sigma(Re)$ for $1 \leqslant Re \leqslant 200$ in the form

$$\sigma(Re) = 0.04+0.664Re^{0.077}. \quad (41)$$

By substituting asymptotics (39), in view of equation (41), into equation (38) at $Sh_0 = 1$, the approximate expression for the mean Sherwood number is obtained which differs from equation (40) for $1 \leqslant Re \leqslant 200$ and $1 \leqslant Pe \leqslant 2000$ at most at $Re = 200$ and $Pe = 2000$ by less than 6%. The results of calculations by equations (38) and (40) at $Re = 1$ and 200 are presented in Fig. 1. Emphasis should be placed upon the fact that relations (38), (39) and (41) ensure a correct result for the limiting case $Pe = 0$, whereas the error of equation (40) at $Re = 200$ reaches here 30%.

The comparison performed in Sections 3 and 4 clearly reveals that the region of applicability of equation (38) is much wider than that of the original relation (36).

5. DIFFUSION TO A SPHERICAL BUBBLE AT DIFFERENT REYNOLDS NUMBERS

The problem of mass exchange of a spherical bubble with a laminar translational flow at different Reynolds

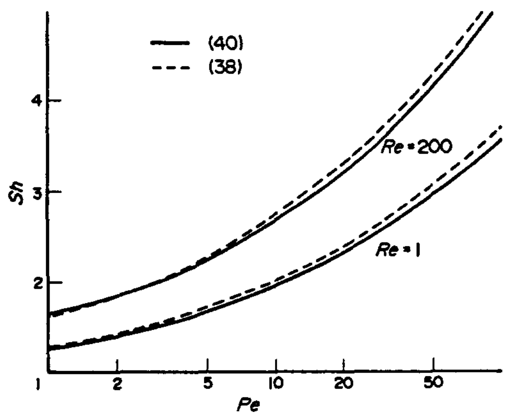

FIG. 1. Sherwood number vs Peclet number for a solid spherical particle in a translational flow at intermediate Reynolds numbers.

numbers was investigated numerically in ref. [11]. The computational results for the mean Sherwood number in the limiting cases of small and large Reynolds numbers are presented in Fig. 2(a).

Let the equation for the Sherwood number be constructed by the asymptotic analogy method. As in the original relation, use will be made of the expression

$$Sh = 0.6+(0.16+0.21Pe)^{1/2} \tag{42}$$

which approximates well the numerical results of the solution for $Re \approx 0$.

At large Peclet numbers, equation (42) yields the asymptotics

$$Sh_\infty = 0.46\sqrt{(Pe)} \quad (Pe \to \infty) \tag{43}$$

which coincides with the analytical result obtained for the corresponding problem in the diffusional boundary layer approximation [3]. When $Pe \to 0$, equation (42) gives $Sh_0 = 1$.

The Peclet number from equation (43) will be expressed in terms of Sh_∞ and placed into equation (42), thereby resulting in

$$Sh = 0.6+(0.16+Sh_\infty^2)^{1/2}. \tag{44}$$

This formula can be derived conventionally by setting $A = 1$, $B = 0.46$, $n = 0$, $m = 1/2$ in equations (7) and taking into consideration that the form of the function F is dictated by the right-hand side of relation (42). Next the equality $Sh_0 = 1$ should be used which is valid for spherical bubbles at any Re.

Equation (44) can already be employed for predicting the mean Sherwood number per spherical bubble at moderate and large Reynolds numbers. The value of Sh_∞ corresponding to $Pe \gg 1$ depends in this case on the Reynolds number. In particular, when $Re \to \infty$, it should be set in equation (44) [11] that

$$Sh_\infty = (2Pe/\pi)^{1/2}. \tag{45}$$

Figure 2(b) presents the results of comparison of approximate relation (44) with the numerical data for the limiting cases $Re \to 0$ and $Re \to \infty$. It is evident that the maximum error amounts to about 6%.

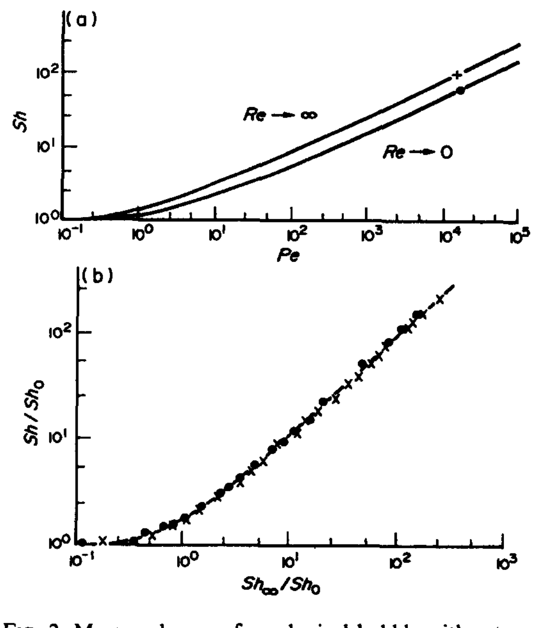

FIG. 2. Mass exchange of a spherical bubble with a translational flow at small and large Reynolds numbers: (a) Sherwood number vs Peclet number; (b) Sherwood number vs asymptotic coordinate. ● ● ● ● ●, $Re \to 0$ [11]; × × × × ×, $Re \to \infty$ [11]; ————, constructed following formula (44).

6. UNSTEADY DIFFUSION TO A DROPLET, BUBBLE AND SOLID PARTICLES IN TRANSLATIONAL AND SHEAR FLOWS

Attention will be focused on unsteady mass transfer to the surface of a solid spherical particle (droplet, bubble) of radius a in a developed flow. It is assumed that at the initial time instant $t = 0$ concentration in the continuous phase is uniform and equal to C_∞, whilst when $t > 0$, a complete absorption of the substance dissolved in the liquid proceeds on the particle surface.

In the spherical coordinate system r, θ, φ with the origin fixed at the particle centre, the corresponding non-stationary problem on distribution of the concentration is represented by the convective diffusion equation with the initial and boundary conditions which in dimensionless variables have the form

$$\frac{\partial c}{\partial \tau} + Pe\,(\mathbf{v}\cdot\nabla)c = \Delta c \tag{46}$$

$$\tau = 0, c = 0; \quad y = 1, c = 1; \quad y \to \infty, c \to 0 \tag{47}$$

where $c = (C_\infty - C)/C_\infty$, $Pe = aU/D$, $y = r/a$, $\tau = Dt/a^2$, U is the characteristic flow velocity, D the diffusion coefficient and Pe the Peclet number. The liquid velocity field $\mathbf{v}$ is assumed to be assigned and stationary.

For a spherical bubble the dependence of the mean Sherwood number on time in an axisymmetric linear shear Stokes' flow was obtained in ref. [4] in the diffusion boundary layer approximation

$$Sh = \left[\frac{3Pe}{\pi}\coth(3Pe\,\tau)\right]^{1/2} \tag{48}$$

where $Pe = a^2E/(2D)$, E is the shear coefficient.

Following the suggested method, relation (48) can be rewritten using asymptotic coordinates. To this end, the leading terms of the Sherwood number expansions, equation (48), can be found at small and large dimensionless times

$$Sh_0 = \left(\frac{1}{\pi\tau}\right)^{1/2} \quad (\tau \to 0)$$

$$Sh_\infty = \left(\frac{3Pe}{\pi}\right)^{1/2} \quad (\tau \to \infty). \tag{49}$$

Asymptotics (49) coincide with equation (7) at $A = \pi^{-1/2}$, $B = (3Pe/\pi)^{1/2}$, $n = -1/2$, $m = 0$ accurate to the evident translation into the new notation $Sh \to x$. Substituting these values into formula (9) and taking into account that the explicit form of the function F is prescribed by expression (48) gives the sought relation

$$\frac{Sh}{Sh_\infty} = \left[\coth\left(\frac{Sh_\infty^2}{Sh_0^2}\right)\right]^{1/2}. \qquad (50)$$

Formula (50) can also be derived in a different fashion. For this, the quantities τ and Pe from equations (49) are expressed in terms of the quantities Sh_0 and Sh_∞ thereby yielding $\tau = 1/(\pi\, Sh_0^2)$ and $Pe = \pi\, Sh_\infty^2/3$. By substituting these expressions into equation (48), relation (50) is obtained.

Now it will be shown that expression (50) can be used with success for an approximate prediction of the rate between droplets, particles and bubbles and various flows at large Peclet numbers.

First it should be noted that at low values of τ the second term on the left-hand side of equation (46) can be neglected and the Laplacian on the right-hand side of this equation can be approximately replaced by $\partial^2 c/\partial y^2$. The solution of the corresponding 'shortened' equation is expressed in terms of the complementary error function and leads to the asymptotics for the Sherwood number $Sh_0 = (\pi\tau)^{-1/2}$ which is valid for the entire class of problems under consideration. Therefore, instead of equation (50) it is possible to write [1]

$$\frac{Sh}{Sh_\infty} = [\coth(\pi\, Sh_\infty^2 \tau)]^{1/2} \qquad (51)$$

where $Sh_\infty = Sh_\infty(Pe)$ is the Sherwood number for the developed diffusional regime.

Comparison of the approximate formula obtained by substituting the corresponding stationary value [3]

$$Sh_\infty = \left[\frac{2Pe}{3\pi(\beta+1)}\right]^{1/2}, \quad Pe = \frac{aU_\infty}{D} \qquad (52)$$

into equation (51) with the results of refs. [12–14] shows that in the case of unsteady mass exchange of a spherical droplet with the developed Stokes flow (U_∞ is the liquid velocity at a distance from the droplet, β the droplet and the surrounding liquid viscosity ratio) the maximum error of expressions (51) and (52) is less than 1% (see Fig. 3).

It should be noted that the results of refs. [12–14] obtained for the mean Sherwood number are given as a fairly complex integral which cannot be represented in a simple analytical form such as equation (51).

Placing into equation (51) the quantities [3]

$$Sh_\infty = 0.624 Pe^{1/3}, \quad Pe = aU_\infty/D \qquad (53)$$

results in the approximate formula for predicting the Sherwood number in the case of unsteady mass transfer to a solid spherical particle in a developed translational Stokes flow. The maximum difference between expressions (51), (53) and the approximation

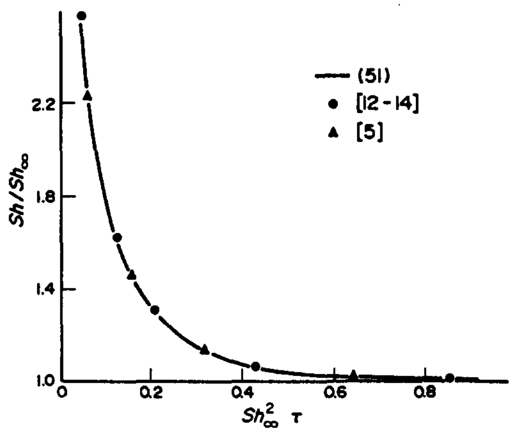

FIG. 3. Comparison of approximate formula for the Sherwood number (51) with the results obtained for a translational flow around a spherical droplet [12–14] and a solid particle [5].

[5] of the numerical–analytical results [15] for the problem of interest comprises less than 2% (Fig. 3).

Comparison with the solution of the three-dimensional non-stationary problem on diffusion to a spherical droplet in a plane shear flow [16] demonstrates that the error of formula (51) (where $Sh_\infty = 0.615\sqrt{(Pe)}$ [4]) does not exceed 1.8%.

Table 2 provides the outcome of comparison of the results of computation for the mean Sherwood number made by formula (51) with the available data for different cases of flow around spherical droplets, bubbles and solid particles at large Peclet numbers.

It should be marked that expression (50) ensures a correct asymptotic result at short and long times and can be used to estimate the unsteady mass transfer rate for non-spherical particles, droplets and bubbles when $Pe \gg 1$. It will be shown in Section 9 that formula (50) is also good for predicting a wide class of more complex non-linear problems of the unsteady diffusion boundary layer.

7. MASS EXCHANGE OF DROPLETS AND PARTICLES WITH FLOW IN THE PRESENCE OF VOLUMETRIC CHEMICAL REACTION

Consideration will be given to steady convective mass exchange of a spherical particle (droplet) of radius a with an incompressible liquid in the presence of the first-order volumetric chemical reaction. The process of the reagent transfer in the continuous phase is described in terms of dimensionless variables by the following equation and boundary conditions:

$$Pe\,(\mathbf{v}\cdot\nabla)\bar{c} = \Delta\bar{c} - k_v\bar{c} \qquad (54)$$

$$y = 1,\ \bar{c} = 1; \quad y \to \infty,\ \bar{c} \to 0 \qquad (55)$$

where $\bar{c} = C/C_s$, $k_v = a^2 K_v/D$, C_s is the particle surface concentration, K_v the volumetric chemical reaction rate constant (the remaining notation was given previously in Section 6).

For solving the stationary problems on convective

Table 2. Maximum error of formula (51) for different flows around spherical particles, droplets and bubbles

No	Type of particles	Type of flow	Method of solution	Error of formula (51) (%)	Reference
1	Droplet, bubble	Axisymmetric shear Stokes flow	Analytical, DBLA†	0 0	[4]
2	Droplet, bubble	Translational Stokes flow	Analytical, DBLA	0.7	[12–14]
3	Droplet, bubble	Three-dimensional Stokes flow	Analytical, DBLA	1.8	[16]
4	Bubble	Laminar translational flow at large Reynolds numbers	Analytical, DBLA	0.7	[13, 14]
5	Bubble	Axisymmetric shear flow at large Peclet numbers, ref. [17]	Analytical, DBLA	0	The present study
6	Droplet, bubble	Flow induced by electric field	Analytical, DBLA	0	[18]
7	Particle	Translational flow of ideal (inviscid) liquid	Analytical, DBLA	0.7	[13, 14]
8	Solid particle	Translational Stokes flow	Interpolation of numerical–analytical results [15]	1.4	[5]
9	Solid particle	Translational Stokes flow	Finite-differential method (at $Pe = 500$)	4	[6]

†DBLA, diffusional boundary-layer approximation.

mass exchange of droplets and particles with the liquid in the presence of the first-order volumetric chemical reaction it is practical to use the results of solutions for the corresponding non-stationary problems without volumetric reaction. In fact, upon applying to equation (46) and boundary conditions (47) the Laplace–Carson transformation (with the valid parameter k_v)

$$\bar{c} = \mathbf{L}^* c \equiv k_v \int_0^\infty \exp(-k_v \tau) c \, d\tau \qquad (56)$$

the problem with volumetric chemical reaction, equations (54) and (55), is arrived at.

It follows from equation (56) that the Sherwood number Sh corresponding to the solution for the problem with the first-order volumetric reaction, equations (54) and (55), can be expressed in terms of the auxiliary Sherwood number (found by solving the non-stationary problem (46), (47)) as follows:

$$\overline{Sh} = \mathbf{L}^* Sh.$$

A useful estimate will be obtained which will be needed for what follows. Let Sh be the mean Sherwood number corresponding to the exact solution of the auxiliary problem, equations (46) and (47), and Sh_{ap} the approximate expression for the Sherwood number with the error equal to ε, i.e.

$$|Sh - Sh_{ap}| \leqslant \varepsilon. \qquad (57)$$

By applying the Laplace–Carson transformation, with allowance for relation (57), to the difference $Sh - Sh_{ap}$, the following inequality is obtained:

$$\overline{Sh} - \overline{Sh}_{ap} = \mathbf{L}^*(Sh - Sh_{ap}) \leqslant \varepsilon \mathbf{L}^* 1 = \varepsilon$$

where $\overline{Sh}$ and $\overline{Sh}_{ap}$ are the exact and appropriate values of the Sherwood number corresponding to the solution for the non-stationary problem with the first-order volumetric chemical reaction, equations (54) and (55). In much the same way it is found that $\overline{Sh} - \overline{Sh}_{ap} \leqslant \varepsilon$. Therefore, the following inequality is valid:

$$|\overline{Sh} - \overline{Sh}_{ap}| \leqslant \varepsilon. \qquad (58)$$

The estimate (58) shows that, having obtained a sufficiently good approximate relation for the auxiliary Sherwood number in the non-stationary problem by the Laplace–Carson transformation, it is possible to get a satisfactory relation (of the same accuracy) for the mean Sherwood number in the stationary problem with the first-order volumetric chemical reaction.

With the aforegoing considered, use will now be made of the results obtained in Section 6 which treated the non-stationary diffusion boundary layer problems. Expression (51) will be employed as the auxiliary mean Sherwood number.

Applying the Laplace–Carson transformation to equation (51) gives the approximate solution for a number of corresponding stationary problems, equations (54) and (55), with the first-order volumetric chemical reaction [19]:

$$\frac{\overline{Sh}}{\overline{Sh}_0} = \Phi\left(\frac{k_v}{\overline{Sh}_0^2}\right) \qquad (59)$$

where the function Φ is assigned by the integral

$$\Phi(x) = x \int_0^\infty \exp(-x\rho)[\coth(\pi\rho)]^{1/2} \, d\rho. \qquad (60)$$

In equation (59) the quantity $\overline{Sh}_0$ corresponds to the mean Sherwood number in the absence of volumetric chemical reaction, i.e. at $k_v = 0$. In forming expression (59), account was taken of the equality $Sh_\infty = \overline{Sh}_0$, in which the quantity

$$Sh_\infty = \lim_{\tau \to \infty} Sh$$

complies with the developed diffusional regime in the non-stationary problem, equations (46) and (47), and

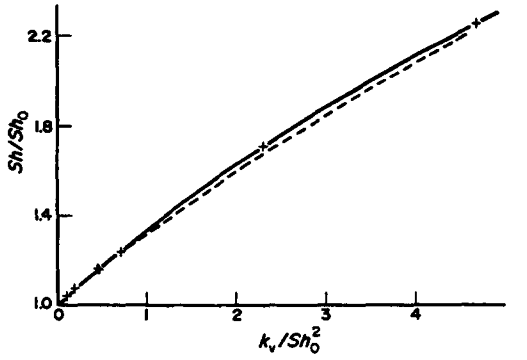

FIG. 4. Sherwood number vs dimensionless volumetric chemical reaction rate constant for an axisymmetric shear flow around a spherical droplet: ————, formula (59); — — — —, formula (61); × × × × ×, data of ref. [20].

$$\overline{Sh}_0 = \lim_{k \to 0} \overline{Sh}$$

gives the mean Sherwood number in problem (54) and (55) at $k_v = 0$.

Relation (59) could also be derived in a different way by applying the Laplace–Carson transformation to equation (48) with the subsequent calculation of the asymptotics $\overline{Sh}$ for $k_v \to 0$ and $k_v \to \infty$ with the results obtained in Section 1. It should be taken into account that for spherical droplets and particles $\overline{Sh}_\infty = \sqrt{k_v}$.

Approximate relation (59) adequately reflects the structure of the dependence of the Sherwood number on the complex $k_v/\overline{Sh}_0^2$ at large Peclet numbers (in the diffusion boundary layer approximation) and provides a correct asymptotic result for $k_v \to 0$ and $k_v \to \infty$. Relations (59) and (60) are shown in Fig. 4 by a solid line. The dots correspond to the solution for the problem of mass exchange of a spherical droplet with the translational Stokes flow obtained [20].

It is significant to note that equations (59) and (60) for the axisymmetric shear flow around a spherical droplet correspond to the exact solution of the diffusion boundary layer equation. The maximum error of expression (59) for some other cases of mass exchange of spherical droplets, bubbles and solid particles with various flows can be evaluated using the last but one column of Table 2. In particular, it follows from Table 2 that the solution for the three-dimensional problem of diffusion to a spherical droplet in a plane shear flow leads to the relation for the mean Sherwood number which differs from equations (59) and (60) by less than 1.8%.

For approximate calculations of the Sherwood number use can be made of the simple expression

$$\overline{Sh} = \sqrt{k_v} \coth\left(\frac{\sqrt{k_v}}{\overline{Sh}_0}\right) \tag{61}$$

which differs from the more complex equations (59) and (60) by less than 2%. Relation (61) is depicted in Fig. 4 by a dashed line.

The results obtained show that relations (59) and (61) can be used with success to predict approximately the Sherwood number in the problems concerning mass exchange of spherical droplets, particles and bubbles with various flows in the presence of the first-order volumetric chemical reaction at large Peclet numbers. (Recall that the parameter $\overline{Sh}_0$ corresponds to the Sherwood number in similar more simple problems without volumetric reaction at $k_v = 0$.) The Sherwood number for non-spherical droplets and particles can be computed by equations (59) and (61) in which the parameter k_v should be replaced by $\overline{Sh}_\infty^2$.

8. MASS EXCHANGE OF PARTICLES AND DROPLETS WITH A FLOW IN THE PRESENCE OF A SURFACE CHEMICAL REACTION

In this section, more complex non-linear problems of convective mass exchange between solid particles (droplets) and a laminar flow will be investigated. Assume that the concentration at a distance from the particle is constant and equal to C_∞ and that on the interface there is chemical reaction proceeding at the rate $W_s = K_s F_s(C)$ where K_s is the surface reaction rate constant, the function F_s, which is due to the reaction kinetics, meet the requirements $F_s(0) = 0$.

The corresponding boundary-value problem on the concentration distribution in the continuous phase is formulated as

$$Pe(\mathbf{v}\cdot\nabla)c = \Delta c \tag{62}$$

$$y = 1, \quad \frac{\partial c}{\partial y} = -k_s f_s(c) \tag{63}$$

$$y \to \infty, \quad c \to 0. \tag{64}$$

Here, the dimensionless functions and parameters are associated with the original dimensional quantities by the relations

$$c = \frac{C_\infty - C}{C_\infty}, \quad y = \frac{r}{a}, \quad Pe = \frac{aU}{D},$$

$$k_s = \frac{aK_s F_s(C_\infty)}{DC_\infty}, \quad f_s(c) = \frac{F_s(C)}{F_s(C_\infty)}$$

and the notation described in Section 6 is adopted. In particular, for the κ-order reaction $F_s = C^\kappa$ and $f_s = (1-c)^\kappa$.

Generally, it is not difficult to verify that the function f_s possesses the properties

$$f_s(1) = 0, \quad f_s(0) = 1. \tag{65}$$

Now, an approximate formula will be constructed for the mean Sherwood number, with the surface reaction rate being arbitrarily dependent on concentration, taking as the point of departure the most simple case of a quiescent medium which complies with $Pe = 0$ in equation (62).

At $Pe = 0$ the solution of equation (62) which satisfies the condition of attenuation at infinity, equation (64), has the form

$$c = q/y \tag{66}$$

where the unknown constant q is related, according to equation (35), to the Sherwood number as

$$q = Sh. \tag{67}$$

Placing expressions (66) and (67) into the boundary condition on the particle surface, equation (63) yields the algebraic (transcendental) equation for the Sherwood number

$$Sh = k_s f_s(Sh). \tag{68}$$

Allowing for properties of equation (65), the leading terms of the expansions of Sh will be found from equation (68) at small and large values of the parameter k_s

$$Sh_0 = k_s\ (k_s \to 0); \quad Sh_\infty = 1\ (k_s \to \infty). \tag{69}$$

After the redesignations $Sh \to x$ and $k_s \to \tau$, equation (68) can be written as an implicit relation of the type of formula (10): $G(x, \tau) \equiv x - \tau f_s(x) = 0$. Therefore, use can be made of equation (12) in which, following equations (69), it is to be set that $A = B = n = 1$, $m = 0$ and x be replaced by Sh. As a result, the following equation is obtained:

$$\frac{Sh}{Sh_0} = f_s\left(\frac{Sh}{Sh_\infty}\right). \tag{70}$$

In the general case of $Pe \neq 0$ the treatment of problem (62)–(64) shows that for an arbitrary flow around spherical droplets, particles and bubbles the higher-order term of the Sherwood number expansion is given by the expression $Sh_0 = k_s$ when $k_s \to 0$. The aforesaid allows equation (70) to be rewritten in the following form:

$$Sh = k_s f_s\left(\frac{Sh}{Sh_\infty}\right). \tag{71}$$

Approximate equation (71) can be used with success to predict the mean Sherwood number for an arbitrary flow around spherical droplets, particles and bubbles with any dependence of the surface reaction rate on concentration over the entire range of Peclet numbers: $0 \leqslant Pe < \infty$.

In equation (71) the quantity $Sh_\infty = Sh_\infty(Pe)$ corresponds to the diffusional regime of reaction (i.e. to the limiting case $k_s \to \infty$) and should be determined by solving linear auxiliary problem (62) and (64) with the simplest boundary condition on the interface: $y = 1$ and $c = 1$.

It is shown in ref. [4] that equation (71) allows the finding of three, for the translational Stokes flow, and four, for the arbitrary shear flow, first terms of the Sherwood number asymptotic expansion in small Peclet numbers for any kinetics of the surface chemical reaction.

The adequacy of approximate equation (71) at the intermediate Peclet numbers $Pe = 10, 20, 50$ (to these values there correspond the Reynolds numbers

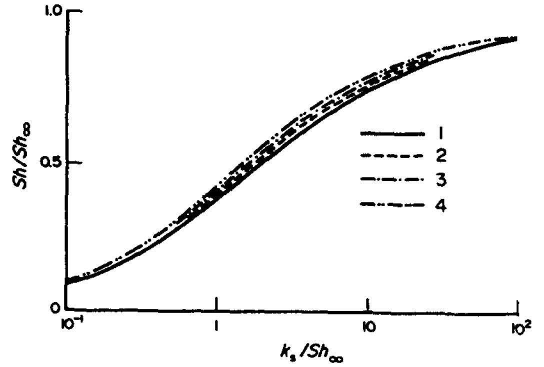

FIG. 5. Sherwood number vs dimensionless second-order surface chemical reaction rate constant. Curve 1 is constructed in accordance with equation (71). Curves 2–4 correspond to a translational flow around a sphere, a circular cylinder and a droplet, respectively.

$Re = 10, 20, 1/2$) in the case of a translational flow around a solid sphere was verified by comparison with the numerical results obtained for the corresponding problem for the first-order surface chemical reaction. It follows from Table 13 in ref. [6] that the error of equation (71) does not exceed here 1.5%.

At large Peclet numbers for the surface reaction of the order $\kappa = 1/2, 1, 2$, the adequacy of equation (71) was tested within the entire range of k_s by comparing its root with numerical results obtained in solving the corresponding integral equations for surface concentration (derived in the diffusion boundary layer approximation) in the case of translational Stokes flow around a sphere, circular cylinder, droplet and bubble [4]. The results of comparison for the second-order reaction ($\kappa = 2$) are plotted in Fig. 5 (for $\kappa = 1/2$ and 1 equation (71) is more exact than for $\kappa = 2$). Curve 1 depicted by a solid line corresponds to the solution of algebraic equation (71) at $f_s(c) = (1-c)^2$. It is seen that the error is observed when $0.5 \leqslant k_s/Sh_\infty \leqslant 5.0$. It does not exceed 6% for a solid sphere (Curve 2), 8% for a circular cylinder (Curve 3) and 12% for a spherical droplet (Curve 4).

It should be noted that in order to calculate the mean Sherwood number for particles of irregular shape, a simpler equation (70) is to be used.

9. UNSTEADY MASS TRANSFER BETWEEN DROPLETS (BUBBLES) AND FLOW WITH AN ARBITRARY DEPENDENCE OF THE DIFFUSION COEFFICIENT ON CONCENTRATION

The analysis is carried out of axisymmetric problems concerning unsteady diffusion to droplets and bubbles at large Peclet numbers with regard for the concentrational dependence of the diffusion coefficient: $D = D(C)$. It is assumed that at the initial time instant the concentration in the surrounding liquid is uniform and equal to C_∞, whereas on the interface there occurs a complete adsorption of the admixture when $t > 0$. Two situations will be analysed simultaneously:

(1) any steady flow, an arbitrarily shaped droplet (bubble),

(2) any unsteady flow, a spherical droplet (bubble).

Use will be made of the ξ, η, λ orthogonal coordinate system fixed with respect to the droplet surface where ξ is directed normal to the surface and η is running along the surface. Without loss of generality it is assumed that the quantity $\xi = 0$ corresponds to the droplet surface and the first component of the metric tensor is equal to unity at $\xi = 0$.

In the diffusion boundary layer approximation the corresponding non-linear non-stationary problem has the form [21]

$$\frac{\partial c}{\partial \tau}+\frac{Pe}{\sqrt{g}}\left(\frac{\partial \Psi}{\partial \xi}\frac{\partial c}{\partial \eta}-\frac{\partial \Psi}{\partial \eta}\frac{\partial c}{\partial \xi}\right)=\frac{\partial}{\partial \xi}\bar{D}(c)\frac{\partial c}{\partial \xi} \quad (72)$$

$$\tau = 0, c = 0; \quad \xi = 0, c = 1; \quad \xi \to \infty, c \to 0 \quad (73)$$

where $c = (C_\infty - C)/C_\infty$, $\tau = tD(C_\infty)/a^2$, $Pe = aU/D(C_\infty)$, $\bar{D}(c) = D(C)/D(C_\infty)$, a is the characteristic droplet (bubble) size, U the characteristic flow velocity. Equation (72) also involves the dimensionless quantities

$$\Psi = \xi\Omega(\tau, \eta), \quad g = g(\eta) \quad (74)$$

which corresponds to the leading terms of the expansions of the stream function and of the third metric tensor invariant at a droplet surface when $\xi \to 0$. To different flows and droplet shapes there correspond different functions Ω and g [4].

The solution to the problem (72)–(74) is sought in the self-similar form [21]

$$c = c\left(\frac{\xi}{\sqrt{\delta}}\right), \quad \delta = \delta(\tau, \eta). \quad (75)$$

Substituting expression (75) into equations (72)–(74) indicates that the unknown function $\delta = \delta(\tau, \eta)$ should satisfy the first-order linear partial differential equation

$$\frac{\partial \delta}{\partial \tau}+\frac{Pe}{\sqrt{g}}\Omega\frac{\partial \delta}{\partial \eta}+2\frac{Pe}{\sqrt{g}}\frac{\partial \Omega}{\partial \eta}\delta = 2 \quad (76)$$

under the initial condition $\delta = 0$ at $\tau = 0$. The concentration profile $c = c(z)$ is obtained by solving the second-order non-linear ordinary differential equation

$$\frac{d}{dz}\left[\bar{D}(c)\frac{dc}{dz}\right]+z\frac{dc}{dz} = 0$$

$$z = 0, c = 1; \quad z \to \infty, c \to 0. \quad (77)$$

Differentiating equation (75) with respect to ξ results in the expression for a local diffusional flux on the droplet surface

$$j = -\left[\bar{D}(c)\frac{\partial c}{\partial \xi}\right]_{\xi=0} = -\frac{1}{\sqrt{\delta}}\left[\bar{D}(c)\frac{dc}{dz}\right]_{z=0}. \quad (78)$$

For the mean Sherwood number corresponding to the solution of non-linear non-stationary problem (72)–(74) the designation $Sh(\bar{D})$ will be used. Taking into consideration that the solutions of equations (76) and (77) are independent, equation (78) will be integrated over the droplet surface. This results in the equality

$$Sh(\bar{D}) = \alpha(\bar{D})Sh(1). \quad (79)$$

Here, $Sh(1)$ is the auxiliary Sherwood number corresponding to linear problem (72)–(74) at the constant diffusion coefficient $\bar{D} = 1$ and $\alpha(\bar{D})$ is the 'non-linearity coefficient' defined by the formula

$$\alpha(\bar{D}) = -\frac{\sqrt{\pi}}{\sqrt{2}}\left[\bar{D}(c)\frac{dc}{dz}\right]_{z=0} \quad (80)$$

where $c = c(z)$ is the solution of equation (77) when $\bar{D} = 1, \alpha = 1$.

It follows from the results of ref. [21] that relation (79) can be extended to the analogous three-dimensional problem of the unsteady diffusion boundary layer. It is also suitable for particles immersed in an inviscid flow (an ideal liquid, a filtrational flow).

Passing in equation (79) to the limit when $\tau \to 0$ and $\tau \to \infty$, the leading terms of the asymptotic expansions of the Sherwood number will be found. Allowing further for the fact that the coefficient α is time-independent, two relations are obtained

$$\frac{Sh(\bar{D})}{Sh_0(\bar{D})} = \frac{Sh(1)}{Sh_0(1)}, \quad \frac{Sh_\infty(\bar{D})}{Sh_0(\bar{D})} = \frac{Sh_\infty(1)}{Sh_0(1)} \quad (81)$$

which are valid for any concentrational dependence of the diffusion coefficient.

The right-hand sides of equalities (81) do not involve the function $\bar{D}$. Therefore, while for linear problem (72)–(74) at $\bar{D} = 1$ the relationship between the asymptotics $Sh_0 \equiv Sh_0(1)$ and $Sh_\infty \equiv Sh_\infty(1)$ is established in the form of equation (6), for non-linear problem (72)–(74) a similar formula is valid

$$\frac{Sh(\bar{D})}{Sh_0(\bar{D})} = f\left(\frac{Sh_\infty(\bar{D})}{Sh_0(\bar{D})}\right) \quad (82)$$

where the function f remains the same for any relation $\bar{D} = \bar{D}(c)$.

The aforegoing means that in the case of axisymmetric shear Stokes flow around a spherical droplet expression (50) can be employed to calculate the mean Sherwood number with an arbitrary concentrational dependence of the diffusion coefficient. It follows from the results obtained in Section 6 that equation (50) is also appropriate for an approximate description of other non-linear non-stationary problems of mass exchange of droplets and bubbles with a flow indicated in Table 2 under Nos. 1–7.

10. DESCRIPTION OF THE METHOD FOR THE ASYMPTOTICS OF AN EXPONENTIAL FORM

The situation will now be considered in which the principal terms of the asymptotic expansions of relation (1) have the form

$$x_0 = a\tau^\alpha(\tau \to 0), \quad x_\infty = b\exp(-\gamma\tau) \quad (\tau \to \infty) \quad (83)$$

where $-1 < \alpha \leqslant 0, \gamma > 0$.

In relations (1) and (83) it is convenient to convert from the quantity τ to the new independent variable z following the formula

$$z = \exp(\gamma\tau) - 1. \quad (84)$$

Here, initial relation (1) takes on the form

$$\bar{x} = \bar{F}(z), \quad \text{where } \bar{F}(z) \equiv F\left(\frac{1}{\gamma}\ln(z+1)\right). \quad (85)$$

Here and hereafter, all the quantities written as functions of the variable z, equation (84), will be denoted by an overbar.

In view of the limiting relations

$$\tau \to 0, z \to \gamma\tau; \quad \tau \to \infty, z \to \exp(\gamma\tau)$$

which follow equation (84), asymptotics (83) will be presented in the form

$$\bar{x}_0 = a\gamma^{-\alpha}z^\alpha \quad (z \to 0); \quad \bar{x}_\infty = bz^{-1} \quad (z \to \infty). \quad (86)$$

It is seen that expressions (86) coincide, accurate to evident redesignations, with the previously considered equation (7) at $A = a\gamma^{-\alpha}$, $B = b$, $n = \alpha$, $m = -1$. Taking into consideration the above and using equation (8), relation (85) will be written with the aid of asymptotic coordinates (86) as

$$\frac{\bar{x}}{\bar{x}_0} = \frac{\gamma^\alpha}{a}\left(\frac{a}{b\gamma^\alpha}\frac{\bar{x}_\infty}{\bar{x}_0}\right)^{\alpha/(\alpha+1)} \times F\left(\frac{1}{\gamma}\ln\left[\left(\frac{b\gamma^\alpha}{a}\frac{\bar{x}_0}{\bar{x}_\infty}\right)^{1/(\alpha+1)}+1\right]\right). \quad (87)$$

Now equation (87) can be employed to approximately calculate the corresponding values for a variety of quantitatively analogous problems. First, for each specific problem the principal terms of the type (83) expansions are to be determined, next the transition to a new variable should be made following equation (84) and lastly the asymptotics should be rewritten in the form of equation (86) and substituted into expression (87). (Recall that it is expedient that the initial function F be obtained, as previously, by solving one of the simplest problems of the class in question.)

In the particular case, when for the entire class of problems considered the equalities $a = 1$, and $\alpha = 0$ are satisfied in accordance with $\bar{x}_0 = 1$, equation (87) simplifies and takes on the form

$$\bar{x} = F\left(\frac{1}{\gamma}\ln\left[\frac{b}{\bar{x}_\infty}+1\right]\right). \quad (88)$$

To illustrate the foregoing statement, the problems on mass exchange of differently shaped bodies, equations (13) and (14), will be tackled once more. In lieu of the mean temperature in equation (15), the analogous quantity will be considered

$$x = 1 - \langle T\rangle \quad (89)$$

which tends to unity when $\tau \to 0$; this corresponds to the values $a = 1$, and $\alpha = 0$ in equations (83).

As before, the heat transfer of a sphere will be considered. Equations (16) and (89) yield that the asymptotics of x at large and small times are described in this case by equations (83) at

$$a = 1, \alpha = 0; \quad b = 6\pi^{-2}, \gamma = \pi^2.$$

Substituting the quantities into equation (88) with allowance for equations (16) and (89) yields the sought relation

$$\bar{x} = \frac{6}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left(\frac{6}{\pi^2}\frac{1}{\bar{x}_\infty}+1\right)^{-n^2} \quad (90)$$

which can already be used to calculate the mass transfer of non-spherical bodies.

For a parallelepiped, it will be found from relations (28) and (89) that the principal term of the expansion of x for $\tau \to \infty$ is prescribed by relation (83) in which

$$b = \left(\frac{8}{\pi^2}\right)^3, \quad \gamma\tau = \frac{\pi^2}{4}\left(\frac{1}{R_1^2}+\frac{1}{R_2^2}+\frac{1}{R_3^2}\right)\chi t.$$

Therefore, following equations (84) and (86) the quantity $\bar{x}_\infty$ can be defined as

$$\bar{x}_\infty = \left(\frac{8}{\pi^2}\right)^3\left\{\exp\left[\frac{\pi^2}{4}\left(\frac{1}{R_1^2}+\frac{1}{R_2^2}+\frac{1}{R_3^2}\right)\chi t\right]-1\right\}^{-1}. \quad (91)$$

Placing expression (91) into equation (90) gives the approximate time dependence of the mean temperature of the parallelepiped.

For a finite cylinder, it follows from equations (30) and (89) that the asymptotics x_∞ of x at $\tau \to \infty$ is determined by equality (83) in which

$$b = \frac{32}{\pi^2\mu_1^2}, \quad \gamma\tau = \left(\frac{\mu_1^2}{R^2}+\frac{\pi^2}{4l^2}\right)\chi t.$$

With these relations taken into consideration the following equation can be obtained with the aid of expressions (84) and (86):

$$\bar{x}_\infty = \frac{32}{\pi^2\mu_1^2}\left\{\exp\left[\left(\frac{\mu_1^2}{R^2}+\frac{\pi^2}{4l^2}\right)\chi t\right]-1\right\}^{-1}. \quad (92)$$

Substituting equation (92) into expression (90) yields the approximate formula for the mean temperature of the finite cylinder.

Now, the accuracy of the constructed relations will

Table 3. Comparison of the exact and approximate values of the mean temperature $\langle T \rangle$ of differently shaped bodies (exponential approximation)

Asymptotic coordinate, $\bar{x}_\infty$	0.2	0.3	0.5	1.0	2.0	5.0	20.0
Sphere, formula (90)	0.151	0.203	0.281	0.402	0.526	0.670	0.823
Parallelepiped, formula (93); $E_i = R_i/R_1$ $(i = 2, 3)$							
$E_2 = 1, \quad E_3 = 0.25$	0.166	0.225	0.312	0.441	0.565	0.702	0.842
$E_2 = 1, \quad E_3 = 0.5$	0.151	0.203	0.282	0.403	0.526	0.669	0.821
$E_2 = 1, \quad E_3 = 1$	0.147	0.196	0.271	0.387	0.509	0.654	0.812
$E_2 = 1, \quad E_3 = 2$	0.151	0.202	0.279	0.398	0.520	0.664	0.818
$E_2 = 1, \quad E_3 = 4$	0.160	0.214	0.294	0.416	0.538	0.679	0.827
$E_2 = 1, \quad E_3 = 4$	0.159	0.214	0.297	0.421	0.545	0.685	0.831
Cylinder, formula (94); $E = R/l$							
$E = 0.25$	0.163	0.219	0.302	0.426	0.550	0.689	0.834
$E = 0.5$	0.154	0.206	0.286	0.407	0.531	0.673	0.824
$E = 1$	0.149	0.199	0.275	0.393	0.515	0.660	0.816
$E = 2$	0.151	0.203	0.281	0.402	0.525	0.668	0.821
$E = 4$	0.162	0.220	0.306	0.434	0.559	0.698	0.839
Droplet, formula (99)	0.155	0.209	0.290	0.413	0.533	0.672	0.824

be estimated. For this, the time t will be eliminated from expressions (28) and (91) and further relation (89) will be taken into consideration. As a result, the expression for the parallelepiped is

$$\bar{x} = \left(\frac{8}{\pi^2}\right)^3 \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sum_{k=1}^{\infty} \frac{1}{(2n-1)^2(2m-1)^2(2k-1)^2} \times \left[\left(\frac{8}{\pi^2}\right)^3 \frac{1}{\bar{x}_\infty} + 1\right]^{-\beta_{nmk}} \quad (93)$$

where

$$\beta_{nmk} = \frac{(2n-1)^2 R_2^2 R_3^2 + (2m-1)^2 R_1^2 R_3^2 + (2k-1)^2 R_1^2 R_2^2}{R_1^2 R_2^2 + R_1^2 R_3^2 + R_2^2 R_3^2}.$$

By rewriting equation (30) in much the same way with the aid of asymptotics (92), the relation for the finite cylinder is

$$\bar{x} = \frac{32}{\pi^2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{1}{\mu_n^2 (2m-1)^2} \left(\frac{32}{\pi^2 \mu_1^2} \frac{1}{\bar{x}_\infty} + 1\right)^{-\varepsilon_{nm}} \quad (94)$$

where

$$\varepsilon_{nm} = \frac{4l^2\mu_n^2 + \pi^2 R^2 (2m-1)^2}{4l^2\mu_1^2 + \pi^2 R^2}.$$

Table 3 (calculations were performed by L. Yu. Yerokhin) presents the results of computation by equations (90), (93) and (94). As previously, a good agreement between the approximate and exact relations is observed for the mean temperature of differently shaped bodies. (It should be noted that, instead of equation (90), the simpler approximate equation (99) can be employed.)

11. UNSTEADY MASS TRANSFER WITHIN A DROPLET IN A STOKES FLOW

Now, consideration is given to the non-stationary problem of convective mass transfer within a spherical droplet of radius a in the presence of the limiting resistance of the dispersed phase. In terms of dimensionless variables, the corresponding problem for the distribution of concentration is formulated as follows:

$$\frac{\partial \tilde{c}}{\partial \tau} + Pe\,(\mathbf{v} \cdot \nabla)\tilde{c} = \Delta \tilde{c} \quad (95)$$

$$\tau = 0, c = 0; \quad y = 1, \tilde{c} = 1 \quad (96)$$

$$\tilde{c} = \frac{C_0 - C}{C_0 - C_s}, \quad Pe = \frac{aU_\infty}{D}, \quad \tau = \frac{Dt}{a^2}$$

where C_0 is the concentration within the droplet at the initial time instant, C_s the concentration on the interface, U_∞ the non-perturbed velocity of the incident flow, D the diffusion coefficient, Pe the Peclet number and $\mathbf{v}$ the dimensionless liquid velocity vector given, for instance, in refs. [5, 6].

At $Pe = 0$ the time dependence of the mean concentration $x = 1 - \langle \tilde{c} \rangle$ is presented by the right-hand side of equation (90), in which $\bar{x}_\infty = 6\pi^{-2} \times [\exp(\pi^2\tau) - 1]^{-1}$. In the other limiting case, when $Pe \to \infty$ (τ is fixed), the distribution of concentration within a droplet is given by the Kronig–Brink equation [22], the numerical solution of which leads to the following expression for the mean concentration [6]:

$$x = \sum_{n=1}^{5} A_n \exp(-\lambda_n \tau) \quad (97)$$

where $A_1 = 0.6831$; $A_2 = 0.0981$; $A_3 = 0.0813$; $A_4 = 0.0618$; $A_5 = 0.0057$; $\lambda_1 = 26.844$; $\lambda_2 = 137.91$; $\lambda_3 = 315.66$; $\lambda_4 = 724.98$; $\lambda_5 = 1205.2$.

The asymptotics of functions (97) for $\tau \to \infty$ are given by equation (83) at $b = 0.6831$; $\gamma = 26.844$. Substituting these values into equations (86) in view of equation (84) gives the asymptotic coordinate

$$\bar{x}_\infty = A_1(e^{\lambda_1\tau}-1)^{-1}$$
$$= 0.6831[\exp(26.844\tau)-1]^{-1}. \quad (98)$$

The substitution of equation (98) into (90) allows the approximate formula for the mean concentration within the droplet at large Peclet numbers to be obtained.

Expression (97) will be rewritten with the aid of equation (98) in the following way:

$$\bar{x} = \sum_{n=1}^{5} A_n \left(\frac{A_1}{\bar{x}_\infty}+1\right)^{-\lambda_n/\lambda_1}. \quad (99)$$

The results computed by equation (99) are presented in the lower line of Table 3. It is seen that the use of equation (90) to calculate the mean concentration gives an error of about 3%. It is therefore to be expected that relations (90) and (99) can be used with success for determining the mean concentration within the droplet at any intermediate Peclet numbers ($0 < Pe < \infty$).

Conclusions. The specific examples considered in the present study show that, for generality, it is expedient to write the final theoretical (and experimental) results in the asymptotic coordinates of the type x/x_0 and x_∞/x_0. Such an approach often makes it possible to obtain universal relations for approximate description of a number of qualitatively similar problems and processes.

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LA METHODE DES ANALOGIES ASYMPTOTIQUES DANS LA THEORIE DU TRANSFERT DE CHALEUR ET DE MASSE ET DANS LE GENIE CHIMIQUE

Résumé—On suggère une technique simple pour construire des relations approchées à large domaine de validité (la même formule peut être employée pour décrire une variété de problèmes qualitativement identiques mais géométriquement différents, par la forme de la surface, la configuration de l'écoulement, etc.). La technique est basée sur la transition entre les variables ordinaires adimensionnelles et les coordonnées spéciales asymptotiques. L'illustration est faite par référence aux nombreux problèmes spécifiques du transfert de chaleur et de masse et de l'hydrodynamique physicochimique. Une comparaison des relations obtenues avec des cas typiques dont on connait les résultats exacts, numériques, approchés ou asymptotiques, montre une bonne précision et une grande aptitude de la méthode. Celle-ci peut aussi être utilisée avec succès dans d'autres domaines des sciences du génie chimique, de la mécanique et de la physique.

DIE METHODE DER "ASYMPTOTISCHEN ANALOGIEN" UND IHRE ANWENDUNG IM GEBIET DES WÄRME- UND STOFFTRANSPORTS UND DER VERFAHRENSTECHNIK

Zusammenfassung—Es wird eine einfache Methode zur Bildung von Näherungslösungen für ein weites Anwendungsgebiet vorgeschlagen. Hierbei kann eine einzige Gleichung verwendet werden, um eine Gruppe von qualitativ identischen Problemen zu beschreiben, die sich geometrisch, d.h. durch die Gestalt der Oberfläche oder des Strömungsfeldes, etc., unterscheiden. Die angewandte Technik basiert auf der Transformation der gewöhnlichen dimensionslosen Variablen in spezielle asymptotische "Koordinaten". Zur Veranschaulichung werden zahlreiche spezielle Probleme aus dem Bereich des Wärme- und Stofftransports und der physikalisch-chemischen Hydrodynamik angeführt. Der Vergleich der gewonnenen Beziehungen mit einer Anzahl von typischen Beispielen, für die bereits analytische, numerische und Näherungslösungen vorliegen, zeigt eine gute Genauigkeit und die Einsatzmöglichkeiten dieser Methode. Sie kann auch erfolgreich in anderen Gebieten der Verfahrenstechnik, Mechanik oder Physik angewendet werden.

МЕТОД АСИМПТОТИЧЕСКОЙ АНАЛОГИИ В ТЕОРИИ МАССО-ТЕПЛОПЕРЕНОСА И ХИМИЧЕСКОЙ ТЕХНОЛОГИИ

Аннотация—Предлагается простой способ построения приближенных зависимостей, обладающих широким диапазоном применимости (одну и ту же формулу можно использовать для описания целого ряда качественно сходных задач, отличающихся геометрическими факторами—формой поверхности, структурой течения и т. п.). Метод основан на переходе от обычных безразмерных переменных к специальным асимптотическим координатам. Указанный подход широко иллюстрируется на конкретных задачах теории массо-теплопереноса и физико-химической гидродинамики. Проведенное сопоставление полученных зависимостей с целым рядом характерных случаев, для которых уже имеются необходимые для проверки точные, численные, приближенные и асимптотические результаты показывают хорошую точность и большие возможности метода. Предложенный метод с успехом может использоваться также в других областях химической технологии, механики и физики.